1. (a) By applying conservation of energy to a homogenous, isotropic volume of constant total mass and writing radial coordinates in the form $\mathbf{r}(t) = a(t)\mathbf{r}(t_0)$, define the quantities Ω_m and Ω_k which satisfy

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left(\frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2}\right).$$

Explain what this implies for the beginning and subsequent evolution of the universe and how it is consistent with the existence of the Cosmic Microwave Background radiation.

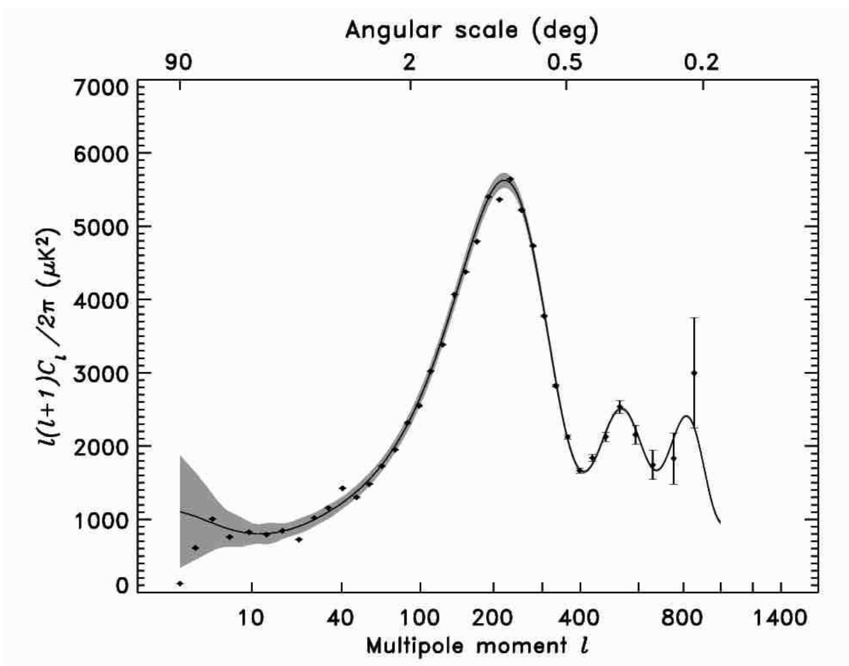
[9]

(b) The points $\mathbf{r}(t_0)$ can be considered *co-moving* points in an expanding space. Find an expression in terms of Ω_m and Ω_k for the total co-moving distance traveled by CMB photons detected today.

[4]

(c) Below is a graph detailing variations in the temperature of these photons. Describe the physical process which produced the peak at $l \approx 200$.

[4]



(d) Studies of gravitaional lensing suggest that $\Omega_m = 0.27$. If the speed of sound-waves in the universe prior to recombination ($a_r = \frac{1}{1000}$) was $\frac{c}{\sqrt{3}}$ and $\Omega_k = 0$, what would be the angular scale of the first peak? Discuss how variations in Ω_k and/or possible modifications to the above equation might improve agreement with the figure.

[8]

2. (a) Find an expression for the central temperature of a star of mass M and radius R which is in Hydrostatic equilibrium. Estimate this value for the sun and discuss fully the implications for energy generation.

[8]

(b) The masses and luminosities of a certain class of stars are found to be approximately related by $L \propto M^3$. Explain the positions of such stars on a logarithmic plot of luminosity against effective temperature (a Hertzsprung-Russell diagram).

[5]

(c) By defining *opacity*, κ, and considering the effects of radiation pressure acting against gravity, deduce that there is a limiting luminosity in this case and discuss, with the aid of an H-R diagram, the subsequent evolution of stars which exceed it.

[12]

3. The number density of nucleons (+) and photons (-) with momentum between p and $p + \mathrm{d}p$ in an isothermal distribution at temperature T is given by

$$\mathrm{d}n(p) = \frac{8\pi}{h^3} \frac{p^2}{\exp(E/kT) \pm 1} \mathrm{d}p$$

(a) Deduce the relation between temperature and total photon energy density and estimate the number density of photons in the present-day universe ($T_{\mathrm{CMB}} = 2.725K$). Define *redshift* and explain its dependance on temperature.

$$\left[\int_0^\infty \frac{x^2}{e^{x^2} - 1} \mathrm{d}x = 2.408 \right]$$

[9]

(b) Find an expression for the ratio of neutrons to protons if they are in thermal equilibrium at $kT << m_p c^2$.

[4]

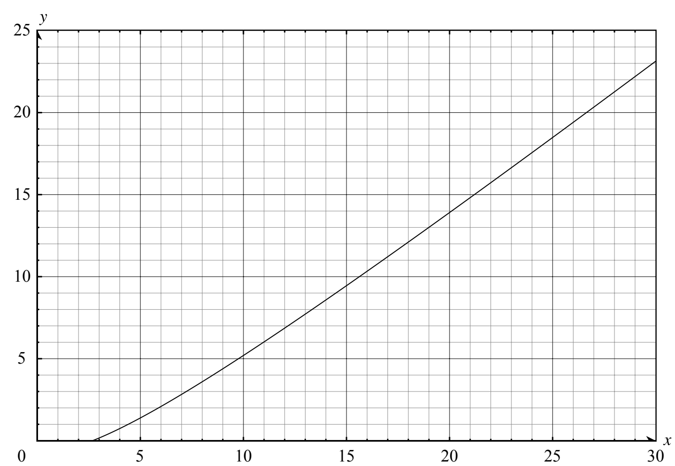
(c) Further such analysis of the synthesis of light elements used in conjunction with measured abundances yields an estimate of the current baryon density: $\Omega_b \approx 0.04$. Show that the this corresponds to a photon-baryon ratio of approximately $10^9 : 1$.

[4]

(d) The CMB photons were last scattered by ionised hydrogen. By finding the temperature at which there is one ionising background photon for every baryon, estimate the redshift of this last-scattering surface. Asses the use of baryon number rather then proton number in this calculation and also the assumption that this transition is instantaneous.

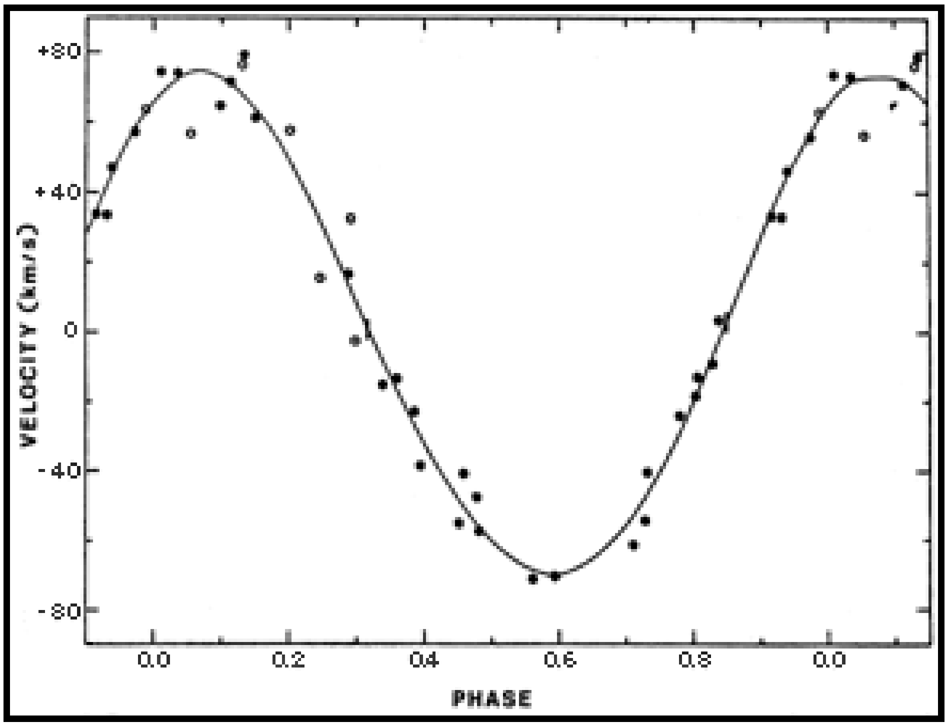
[8]

[The following graph of $y = x - \ln(x^2 + 2x + 2)$ may be of use:]



4. (a) Plotted below is the doppler-inferred velocity of a giant O9 star which has no visible binary partner. Given that the period of this oscillation is 5.6 days and assuming the plane of the orbit is approximately parallel to the line of sight, calculate the radius of the orbit.

[3]



The figure yields a a value for the circular velocity of the star, v_1 of about 75 km/s. Gravitational force and centrepetal force give:

$$\frac{v_1^2}{r_1} = \frac{GM_2}{(r_1 + r_2)^2}$$

(b) The star's spectral type suggests its mass to be of the order of 25 solar masses. Deduce the approximate mass of it's invisible companion.

[8]

(c) By finding the the energy density of fully degenerate fermions, show that the equation of state for material in a neutron star is given by

$$P = \left(\frac{3}{\pi}\right)^{\frac{1}{3}} \frac{hc}{8} \left(\frac{\rho}{m_{\mathrm{H}}}\right)^{\frac{4}{3}}.$$

and explain how this implies an upper limit to the mass of such objects.

[8]

(d) Describe the likley nature of the companion and, with the aid of relevant gravitational calculations, discuss the probable state and evolution of the system.

[6]